

Solving Sequence Problems by Fractal Origami Models

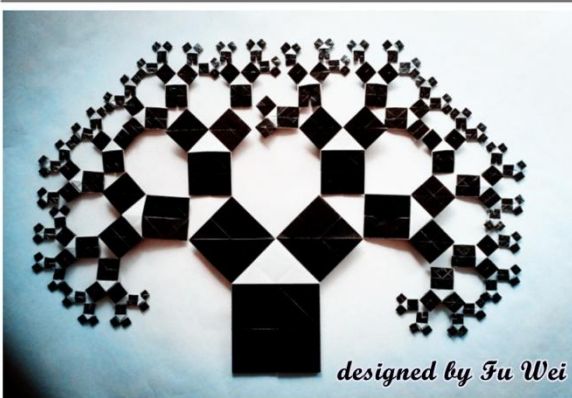
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Abstract

Fractal is a branch in mathematics for describing and simulating naturally occurring objects. Fractals are different from other geometric figures because they scale in a self-similar pattern through expanding symmetry or evolving symmetry. In China, some fractal problems are required to teach in mathematics in senior high schools. It's abstract and hard to understand for some students, so I made some paper models to evoke their interest while practicing their manipulative abilities.

As you know, nearly all sequence problems are related to many items, thus related to recursion, derivation or self similarity. And fractals have such properties, so I managed to fold various paper models including Cube Pile, Checkerboard, Sierpinski triangles, Sierpinski Carpet, Pythagoras Tree, Koch Snowflake, Sphinx Tiling, Quadratic Flake and so on.



Pythagoras Tree Puzzle

From the above picture, we can know the growing process of Pythagoras tree (fractal):

- An isosceles right-angled triangle is derived from a square.
- The right-angle sides of the triangle are connected with two smaller squares.
- By the same token, smaller squares can be obtained.

Suppose altogether 127 squares were obtained, and the side length of the smallest (last) square is 1. Then, what the side length of the largest (first) square?

According to the growing rule of Pythagoras Tree, the numbers of the squares (from large to small) in the puzzle form a geometric sequence:

1, 2, 4, 8, 16, 32, 64, and the sum of the finite sequence are 127.

i.e. $a_n = 2^{n-1} (n \geq 1)$

$$\sum_{i=1}^7 2^{i-1} = 1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$$

The side length of each larger square is $\sqrt{2}$ the side length of its neighbor smaller square. If the side length of the smallest square is 1, then the side lengths of the squares (from small to large) form a new geometric sequence:

1, $\sqrt{2}$, 2, $2\sqrt{2}$, 4, $4\sqrt{2}$, 8, i.e. $b_n = (\sqrt{2})^{n-1} (n \geq 1)$

And the answer to the puzzle is $b_7 = (\sqrt{2})^{7-1} = (\sqrt{2})^6 = (2)^3 = 8$

The areas of the squares (from small to large) form another new geometric sequence:


1, 2, 4, 8, 16, 32, 64 i.e. $c_n = 2^{n-1} (n \geq 1)$

The numbers of the squares (from small to large) form the geometric sequence:

64, 32, 16, 8, 4, 2, 1 i.e. $d_n = 2^{7-n} (n \geq 1)$

So, the area of all the squares is the sum = $c_n * d_n (1 \leq n \leq 7)$

$1*64 + 2*32 + 4*16 + 8*8 + 16*4 + 32*2 + 64*1 = 64*7 = 448$



Answer to Pythagoras Tree Puzzle

Figure 1: Pythagoras Tree Puzzle and Answer

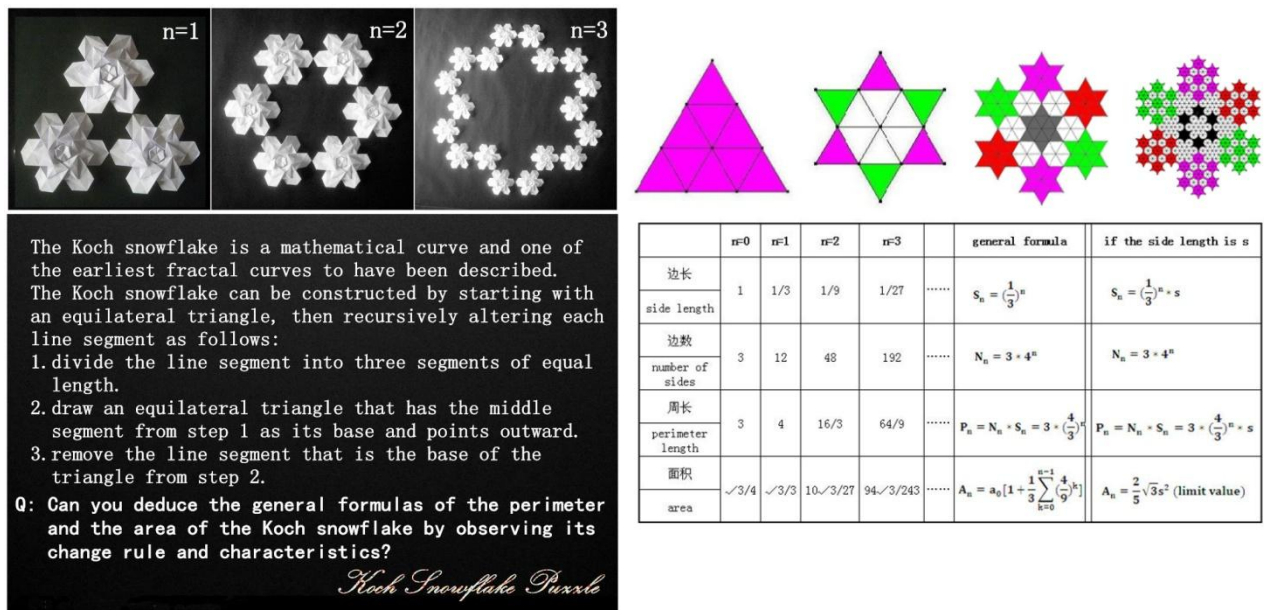


Figure 2: Koch Snowflake Puzzle and Answer

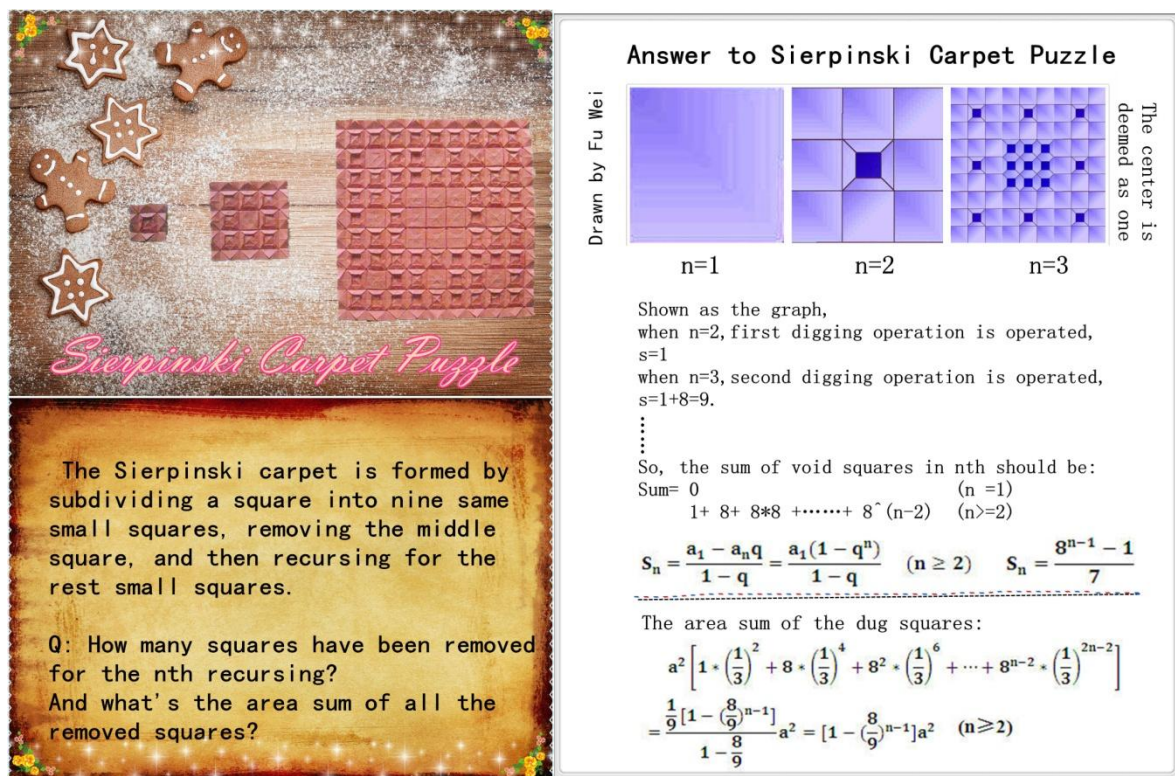


Figure 3: Sierpinski Carpet Puzzle and Answer

During the folding procedure, I must give mature consideration to all aspects of my to-be-folded model. The whole process needs thorough and comprehensive considerations for both math puzzle solving and origami folding, so one can acquire origami design exercises, problem-solving ability improvement, folding skill adeptness and software using flexibility. It's a new way to teach math for kids or teenagers who will be eager to study both math and origami in a playing process while being impressed deeply and happily.